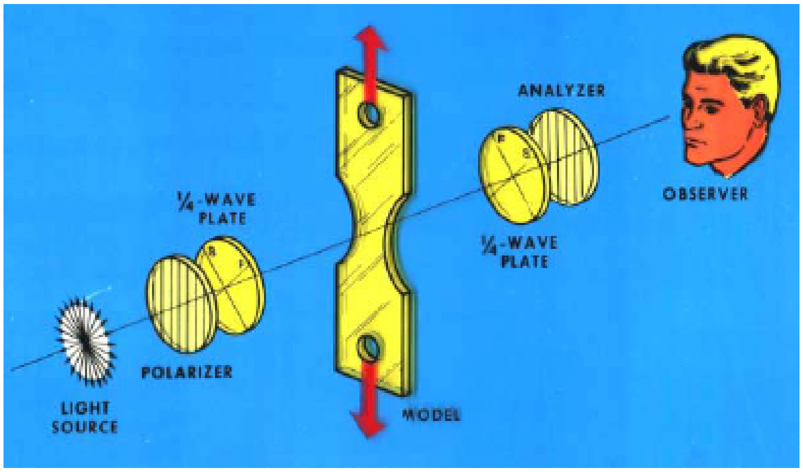
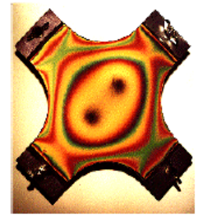
ME 314 - Engineering Design : Mechanical Components
Lecture 9

Note Title

Components of a Polariscope

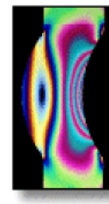


Photoelasticity



Photoelasticity is a visual method for viewing the full field stress distribution in a photoelastic material.

Photoelasticity
(Continued)



When a photoelastic material is strained and viewed with a polariscope, distinctive colored fringe patterns are seen. Interpretation of the pattern reveals the overall strain distribution.

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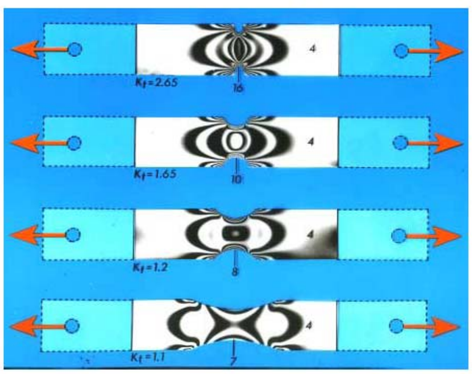
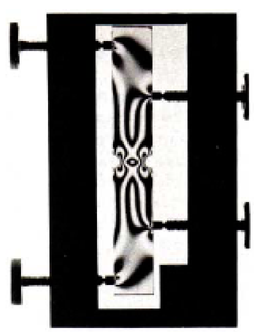
Effect of Discontinuity Geometry



The discontinuity geometry has a significant effect on the stress distribution around it.

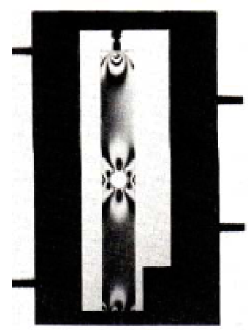
Vishay Lecture-Aid Series, LA-101

Stress Distributions Around Geometric Discontinuities



(c)

Photoelastic fringes in a notched beam loaded in bending.



(b)

Photoelastic fringes in a narrow plate with hole loaded in tension.

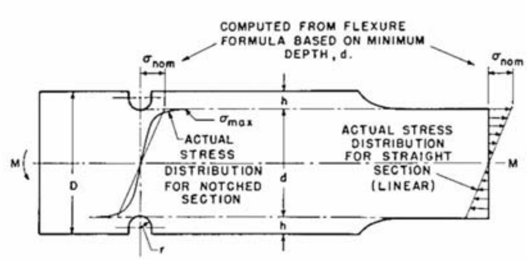


Figure 4-34

Photoelastic Measurement of Stress Concentration in a Flat, Stepped, Notched Bar in Bending

Reproduced from reference 4, Fig. 2, p. 3, reprinted by permission of John Wiley & Sons, Inc.

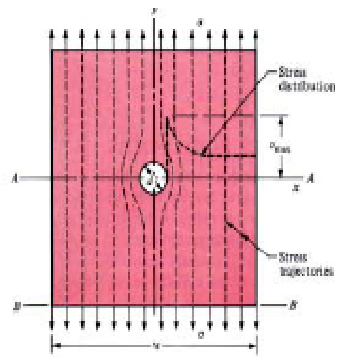
Fig. 4-34 shows the stress concentration caused by notches and fillets in a flat bar subjected to a bending moment. Note that at the right end where the cross-section is uniform, the fringe lines are straight, of uniform thickness, and are equally spaced. This is an indication of a linear stress distribution in this part of the bar (see page 187 of text for more details).

To account for stress concentration, a "**theoretical**," or "**geometrical**" **stress concentration factor, K_t** for normal stress, or K_{ts} for shear stress is defined:

$$\sigma_{max} = K_t\,\sigma_{nom} \qquad , \qquad \tau_{max} = K_{ts}\,\tau_{nom}$$

The nominal stress σ_{nom} (or τ_{nom}) is more difficult to define. Generally, it is the stress calculated by using the stress equations and the net area, or net cross-section (as shown above). But sometimes the gross cross-section is used instead, and so it is always wise to double check your source of K_t and K_{ts}, before calculating the maximum stress.

Geometric Stress Concentration Factors



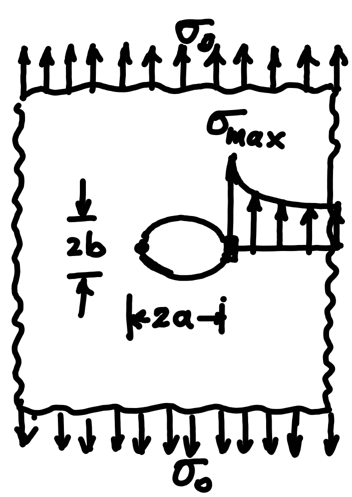
$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

$$\sigma_{nom} = \frac{F}{A_0}$$

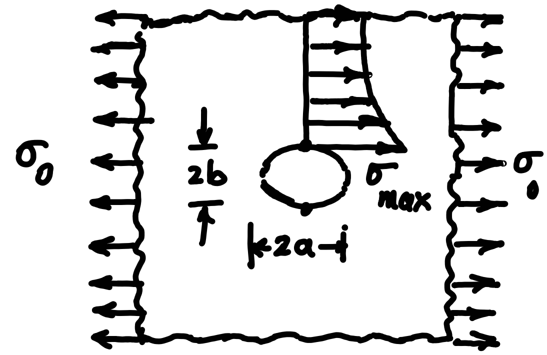
$$A_0 = (w-d)t$$

Geometric stress concentration factors can be used to estimate the stress amplification in the vicinity of a geometric discontinuity.

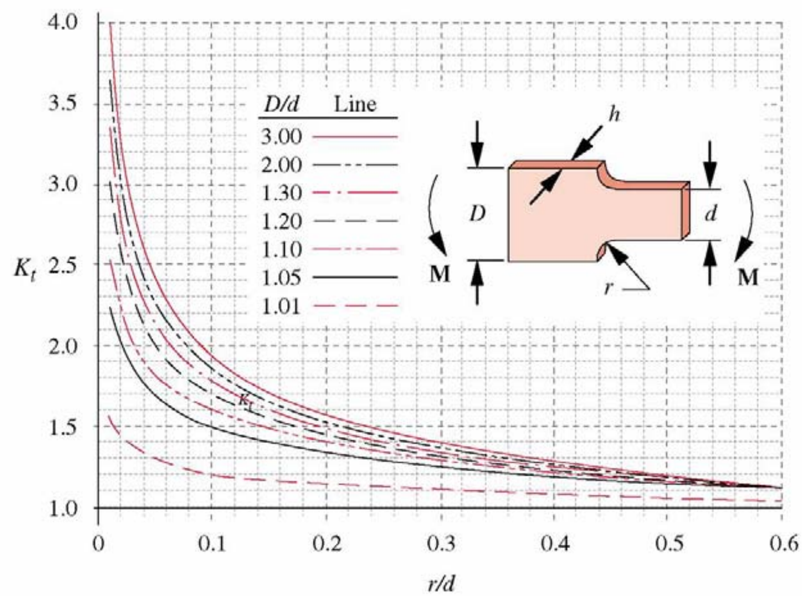
Exact values of K_t are known from Elasticity only for a few shapes, e.g., for an elliptical hole,



When the applied stress is in the direction of major axis of ellipse, we have



For more complicated geometry, experimental methods (e.g., photoelasticity), or computational methods such as FEM are used. The results for many cases are found and are given in the form of graphs such as the ones given in Appendix C of text (pp 999-1006). The most comprehensive collection of stress concentration factor data is the book by Peterson (Peterson, R.E., *Stress Concentration Factors*, Wiley & Sons: NY 1974).



$$\sigma_{nom} = \frac{Mc}{I} = 6\frac{M}{hd^2}$$
$$\sigma_{max} = K_t\sigma_{nom}$$

and :

$$K_t = A\left(\frac{r}{d}\right)^b$$

where :

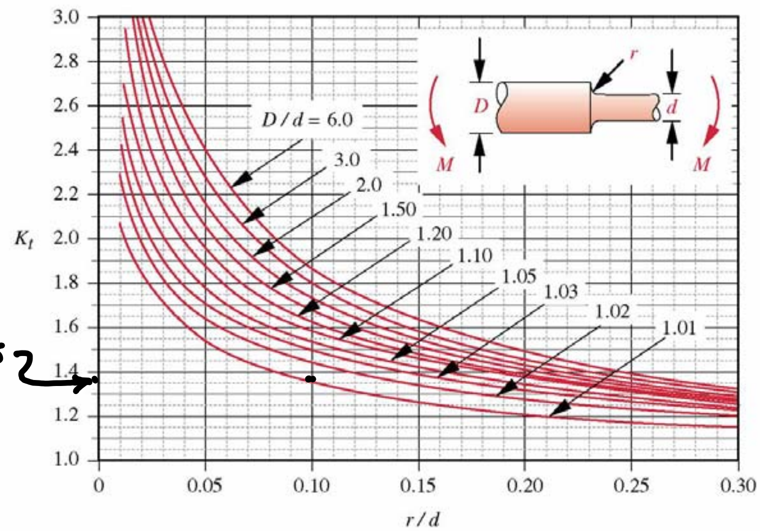
D/d	A	b
3.00	0.907 20	–0.333 33
2.00	0.932 32	–0.303 04
1.30	0.958 80	–0.272 69
1.20	0.995 90	–0.238 29
1.10	1.016 50	–0.215 48
1.05	1.022 60	–0.191 56
1.01	0.966 89	–0.154 17

Figure 4-36

Geometric Stress-Concentration Factors and Functions for a Stepped Flat Bar in Bending - Also see the File APP_E-10

Source: Fig. 73, p. 98, R. E. Peterson, Stress Concentration Factors, John Wiley & Sons, 1975, with the publisher's permission.

$K_t = 1.35$



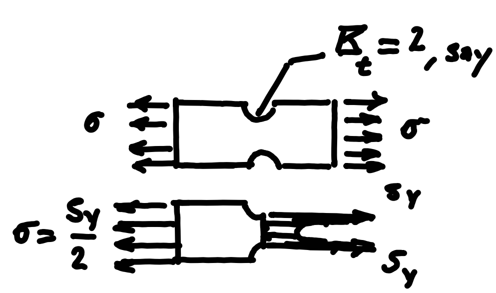
$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
6.00	0.878 68	−0.332 43
3.00	0.893 34	−0.308 60
2.00	0.908 79	−0.285 98
1.50	0.938 36	−0.257 59
1.20	0.970 98	−0.217 96
1.10	0.951 20	−0.237 57
1.07	0.975 27	−0.209 58
1.05	0.981 37	−0.196 53
1.03	0.980 61	−0.183 81
1.02	0.960 48	−0.177 11
1.01	0.919 38	−0.170 32

Figure E-2
Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Bending.

Stress concentration does not always lead to immediate failure. When are we supposed to consider stress concentrations?



*** Stress concentration is not considered for static loading of a material that behaves in a ductile manner. The notched plate shown will undergo local yielding at the stress raiser but it can still hold some load.**

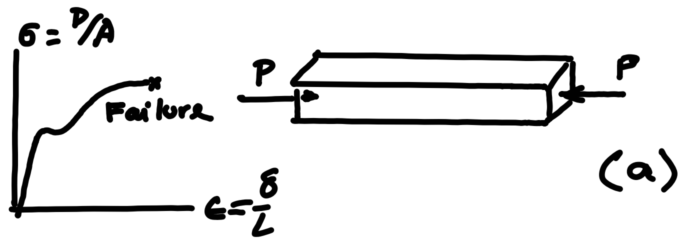
*** Stress concentration should be considered for most materials under dynamic loading (gray cast iron has internal flaws and stress raisers have little effect. These flaws have already been accounted for in measuring their properties).**

*** Stress concentration should be considered, however, for brittle homogeneous materials, e.g., the clear plastic film wrapping of packages.**

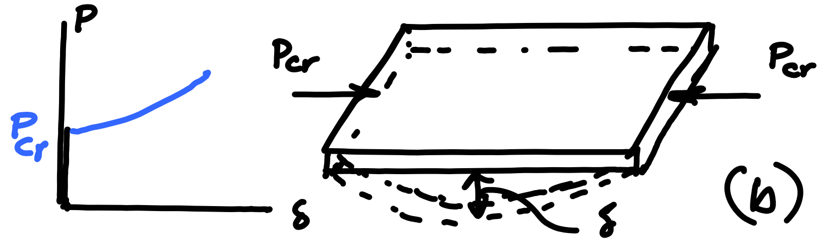
4.16 Axial Compression - Columns

Compression may cause buckling (elastic instability) way before failure due to yield (gradual loss of strength) so it should be checked.

*** Failure due to loss of strength is gradual.**



*** Buckling is sudden with no warning.**



Slenderness Ratio

A *short* column will fail in compression as shown in Fig. (a). An *intermediate*, or *long* column will fail by buckling as shown in Fig. (b). How do we decide whether a column is short, intermediate, or long? The decision is based on the value of a parameter called the *Slenderness Ratio*, S_r,

where ℓ is the length of the column and k is its radius of gyration.

where *I* is the smallest second moment of area of the column's cross section about the neutral axis, and *A* is the cross-sectional area.

Short Columns

Any column with a slenderness ratio, $S_r \leq 10$ is defined to be a short column and the material's yield strength in compression, S_{yc} is used as the limiting factor to compare to the stress $\sigma_x = P/A$.

Long Columns

Euler (1744) derived a formula for P_{cr}:

Note that the only material property that appears in buckling is the stiffness but *not* the material's strength.

Since the second moment of cross-sectional area with respect to the buckling axis (i.e., the smallest value of I about any axis) is given by

where Eq. (4.34) above has been used, Eq. (4.38a) can be written as

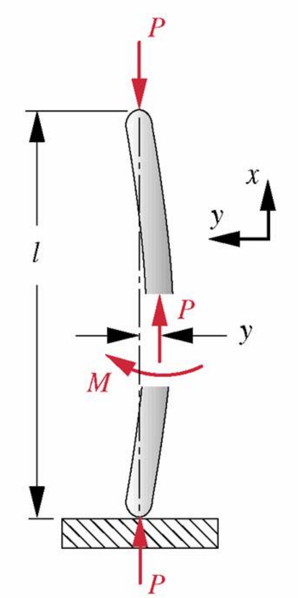
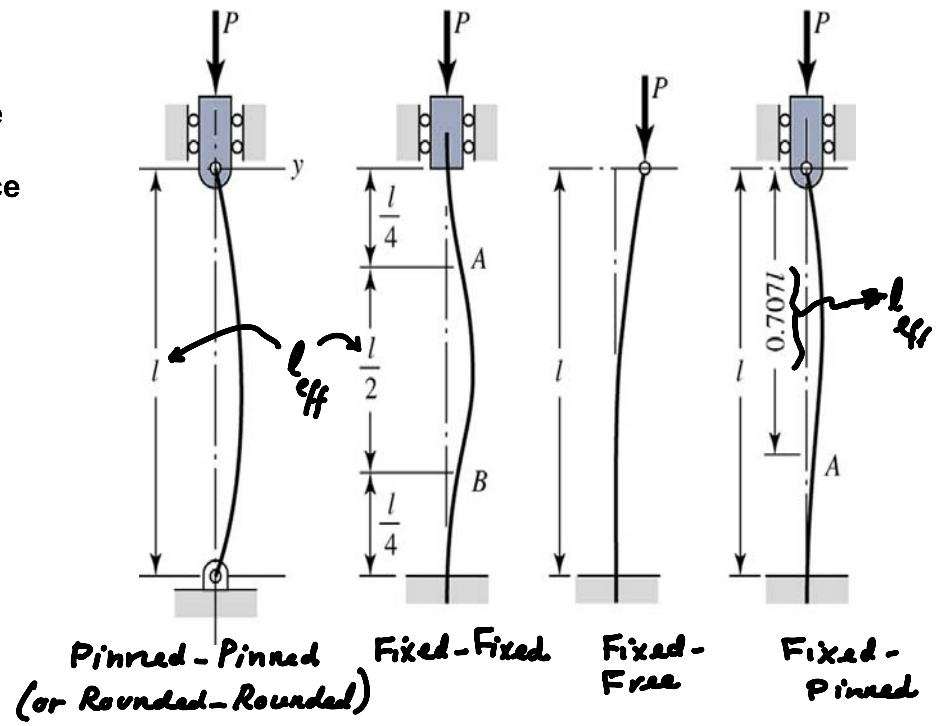


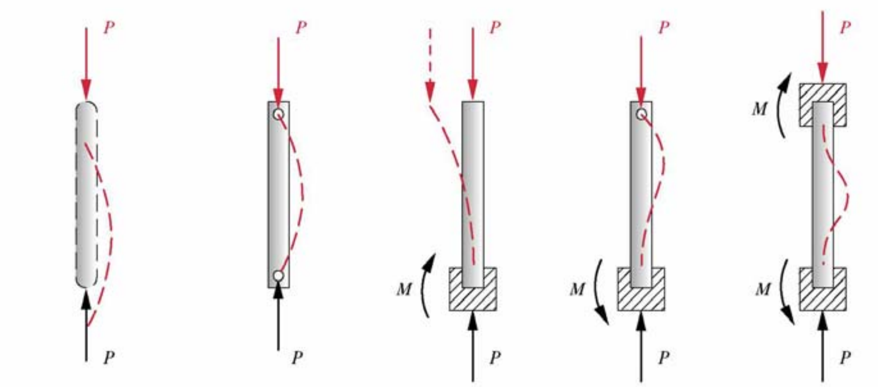
Figure 4-40
Buckling of an Euler Column.

P_{cr}/A is the critical unit load that will cause buckling to occur.

The Euler column formula (4.38) is derived based on the boundary conditions that column ends are rounded-rounded (or pinned-pinned) as in Fig. 4-40 above. It turns out that for other end conditions all we have to do is to modify the definition of the length of the column as shown in the figures below:

Hence, to account for all end conditions, we define an effective length, ℓ_{eff} (see Table 4-4) and replace the slenderness ratio in Eg. (4-35) by





(a) Rounded-rounded (b) Pinned-pinned (c) Fixed-free (d) Fixed-pinned (e) Fixed-fixed

Figure 4-41

Various End Conditions for Columns and Their Resultant Deflection Curves *(Applied Loads Shown in Color - Reactions in Black).*

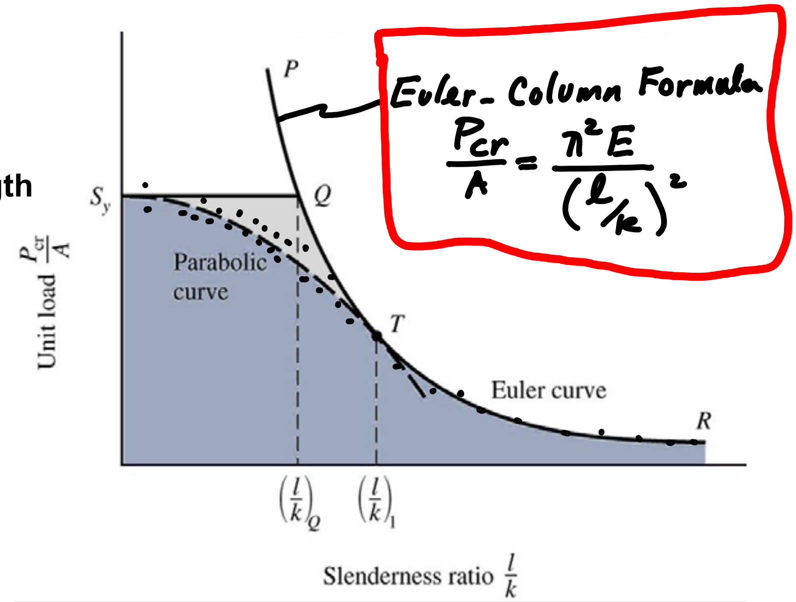
End Conditions	Theoretical Value	AISC* Recommends	Conservative Value
Rounded-Rounded	$l_{eff} = l$	$l_{eff} = l$	$l_{eff} = l$
Pinned-Pinned	$l_{eff} = l$	$l_{eff} = l$	$l_{eff} = l$
Fixed-Free	$l_{eff} = 2l$	$l_{eff} = 2.1l$	$l_{eff} = 2.4l$
Fixed-Pinned	$l_{eff} = 0.707l$	$l_{eff} = 0.80l$	$l_{eff} = l$
Fixed-Fixed	$l_{eff} = 0.5l$	$l_{eff} = 0.65l$	$l_{eff} = l$

Table 4-4

Column End-Condition Effective Length Factors.

Intermediate-Length Columns

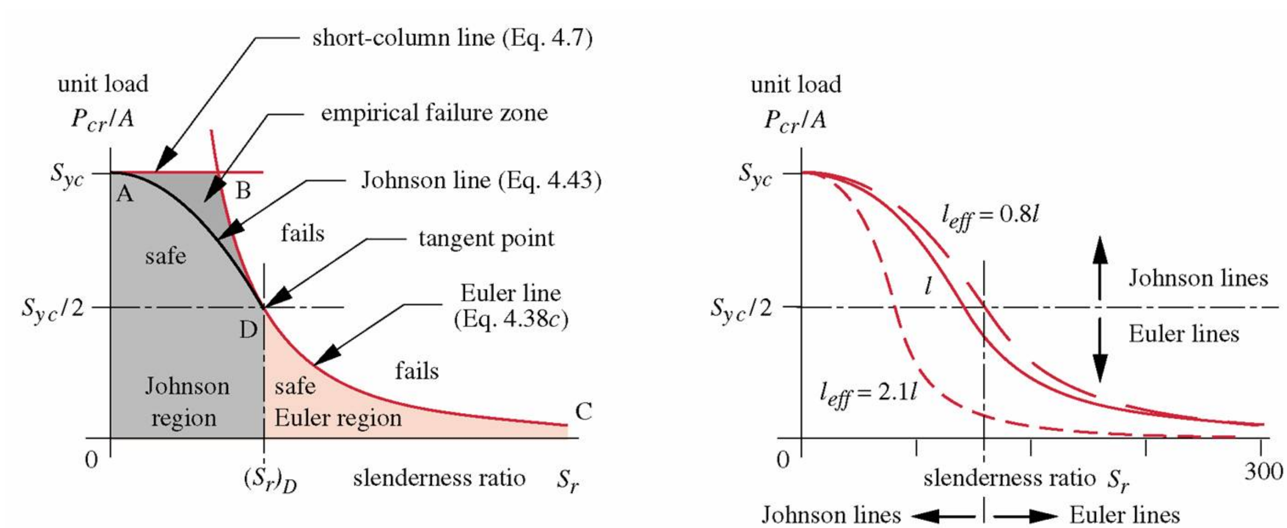
Experimental results show that the Euler formula is applicable only for long columns. For intermediate-length columns, the data seem to follow a parabolic curve so an equation proposed by J. B. Johnson is used:



Euler- Column Formula

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(l/k)^2}$$

Equations (4.38) and (4.43) provide a model for the design of all concentrically loaded columns.



(a) Construction of column failure lines

(b) Failure lines for different end conditions

Figure 4-42
Euler-, Johnson-, and Short-Column Failure Lines.

Please review the formulas for the eccentrically loaded columns on your own.